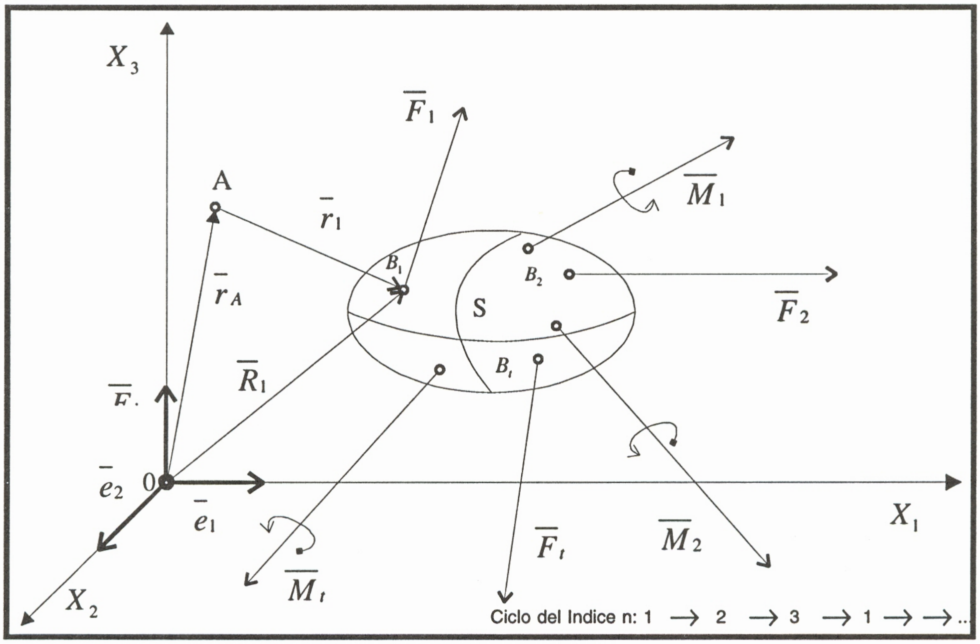
El formalismo matricial en la estática estructural

Por: Johannio Marulanda A.

Generalmente, en la estática, el formalismo matrical se ha limitado al tratamiento del sistema resolutorio de ecuaciones. A continuación el problema de la estática se plantea matricialmente desde un comienzo para lo cual es necesario recuperar la representación vectorial. Su utilidad práctica inmediata reside en la posibilidad de la construcción de un soporte lógico para el uso de computadores.

1. Sean $S \ni B_i$ (solicitados por $\overline{F}_i, \overline{M}_i$) y A en O ($\overline{e}_n$)



2. Factores

$$\overline{F}_i = \sum_{n=1}^{3} F_{ix_n} \overline{e}_n = F_i \sum_{n=1}^{3} \iota_{in} \overline{e}_n = F_i \left| \overline{e}_1 \overline{e}_2 \overline{e}_3 \right| \left\| \iota_1 \iota_2 \iota_3 \right\|_i = \overline{e} F_i \lambda_i$$

$$M_A\left[\overline{F}_i\right] = \overline{r}_i * \overline{F}_i = \left[\overline{R}_i - \overline{r}_A\right] * \overline{F}_i$$

$$\overline{\mathrm{M}}_A = F_i\left[\sum_{n=1}^{3}\Delta x_{in}\overline{e}_n\right] * \left[\sum_{n=1}^{3} l_{in}\overline{e}_n\right], \qquad \Delta X_n = X_n - X_{An}$$

$$= F_i\sum_{n=1}^{3}\left[l_{i(n+2)}\Delta X_{i(n+1)} - l_{i(n+1)}\Delta X_{i(n+2)}\right]\overline{e}_n,$$

(Ciclo: 1, 2, 3, 1 ...),

$$= F_i\left|\overline{e}_1\,\overline{e}_2\,\overline{e}_3\right|\begin{vmatrix} 0 & -\Delta x_3 & \Delta x_2 \\ \Delta x_3 & 0 & -\Delta x_1 \\ -\Delta x_2 & \Delta x_1 & 0 \end{vmatrix}_i \left\|l_1 l_2 l_3\right\| = \overline{e}F_i W_i \lambda_i$$

$$\overline{\mathrm{M}}_i = \sum_{n=1}^{3} M_{ix_n}\overline{e}_n = M_i\sum_{n=1}^{3} l_{in}\overline{e}_n = M_i\left|\overline{e}_1\,\overline{e}_2\,\overline{e}_3\right|\left\|l_1 l_2 l_3\right\|_i = \overline{e}M_i\lambda_i$$

3. Condiciones de equilibrio

$$\sum_{i=1}^{k}\overline{F}_i = \overline{e}\sum_{i=1}^{k}F_i\lambda_i = \overline{e}\Delta_F F = 0$$

$$\sum_{i=1}^{k}\overline{F}_i = \overline{e}\sum_{i=1}^{k}F_iW_i\lambda_i + \overline{e}\sum_{i=1}^{k}M_i\lambda_i = \overline{e}[WF + \Delta_M M] = 0$$

4. Ecuación matricial

$$\mathrm{KQ} = 0$$

donde:

$$K = \begin{vmatrix}\Lambda_F & 0 \\ W & \Lambda_M\end{vmatrix}, \quad Q = \begin{vmatrix}F \\ M\end{vmatrix}, \qquad \Lambda_T = \left|\lambda_1\lambda_2\ldots\lambda_s\right| = |\mathrm{U}|\{\lambda_s\} = J\Lambda_T$$

λ_s - vector de valores modulares

$|\mathrm{U}|$ - Matriz unitaria

$$W = |W_1\lambda_1 W_2\lambda_2 \ldots W_s\lambda_s| = |\mathrm{U}|\{W_s\}\{\lambda_s\} = JW\Lambda_T$$

$$K = \begin{vmatrix} J_F\Lambda_F & 0 \\ J_F W\Lambda_F & J_M\Lambda_M \end{vmatrix} = \left[\begin{vmatrix} J_F & 0 \\ 0 & J_M \end{vmatrix} \begin{vmatrix} U & 0 \\ W & U \end{vmatrix} \begin{vmatrix} \Lambda_F & 0 \\ 0 & \Lambda_M \end{vmatrix}^{\downarrow} \right]^{\downarrow}$$

$\downarrow$: Operación de intercambio de 1.2 y 2.1 de la diagonal

$$W_1 = |\alpha_1\alpha_2\alpha_{31}|_i \quad , \qquad \alpha_{n_i} = I_n\Delta_i = I_n \begin{vmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{vmatrix}_i$$

$$I_1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \bar{1} & 0 \end{vmatrix}, I_2 = \begin{vmatrix} 0 & 0 & \bar{1} \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix}, I_3 = \begin{vmatrix} 0 & 1 & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$W_i = |I_1 I_2 I_3| * \Delta_i = I * \Delta_i$$

$*$: Operación de producto escalar (no conmutativa). El símbolo de matriz opera como paréntesis.

$$W = I * \{\Delta_s\} = I * \Delta$$

$$\begin{vmatrix} U & O \\ W & U \end{vmatrix} = \begin{vmatrix} U & O \\ I * \Delta & U \end{vmatrix} = \begin{vmatrix} U & O \\ I & U \end{vmatrix} * \begin{vmatrix} U & O \\ \Delta & U \end{vmatrix}$$

$$K = \left[\begin{vmatrix} J_F & O \\ O & J_M \end{vmatrix} \left[\begin{vmatrix} U & O \\ I & U \end{vmatrix} * \begin{vmatrix} U & O \\ \Delta & U \end{vmatrix} \begin{vmatrix} \Lambda_F & O \\ O & \Lambda_M \end{vmatrix} \right]^{\downarrow} \right]^{\downarrow} = \left[J\left[(I^\circ * \Delta^\circ)\Lambda \right]^{\downarrow} \right]^{\downarrow}$$

5. Sea el sistema isostático T $\ni t_s$ solicitado por $\overline{F_{io}}, \overline{M}_{io}$ (factores externos) y $\overline{F_{ix}}, \overline{M}_{ix}$ (reacciones).

Para t_S:

$$K_S Q_S = \left| K_{SX} K_{SO} \right| \begin{vmatrix} Q_{SX} \\ Q_{SO} \end{vmatrix} = 0$$

Para T:

$$KQ = \left| K_X K_O \right| \begin{vmatrix} Q_X \\ Q_O \end{vmatrix} = 0$$

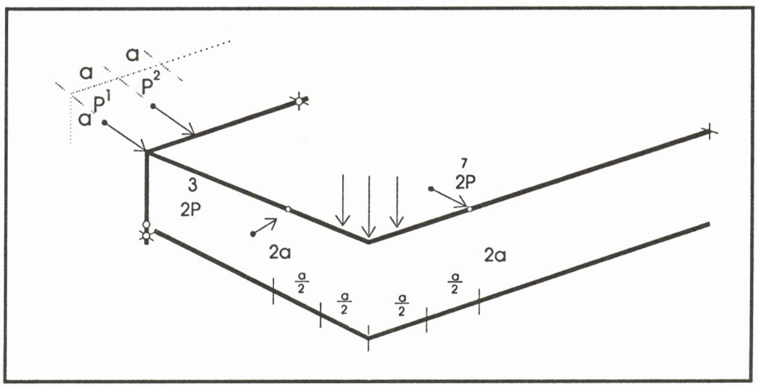
6. Solución

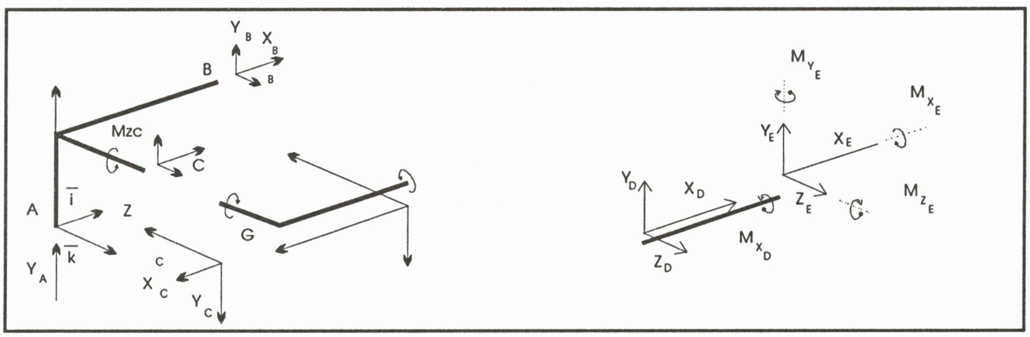
$$KQ = \left| K_X K_O \right| \begin{vmatrix} Q_X \\ Q_O \end{vmatrix} = K_X Q_X + K_O Q_O = O$$

$$Q_X = K_x^{-1} K_o Q_O$$

7. Caso.

7.1 Sea el sistema T y el correspondiente conjunto de cuerpos libres:





7.2 Estructura general de K_X y K_O:

Elemento	Vínculos																		Solicitaciones						
	A	B			C				D				E												
	Y_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	M_{Z_C}	X_D	Y_D	Z_D	M_{X_D}	X_E	Y_E	Z_E	M_{X_E}	M_{Y_E}	M_{Z_E}	1	2	3	4	5	6	7
A B C	X	X	X	X	X	X	X	X											X	X					
CD					X	X	X	X	X	X	X	X									X	X	X	X	X
DE									X	X	X	X	X	X	X	X	X	X							

$$K_X = \left[J_X \left[\left(I^O * \Delta^O{}_X \right) \Lambda_X \right]^{\downarrow} \right]^{\downarrow}, \qquad K_O = \left[J_O \left[\left(I^O * \Delta^O{}_O \right) \Lambda_O \right]^{\downarrow} \right]^{\downarrow}$$

7.3 J_x , J_o. Grafos contruidos a partir de la tabla. En los vínculos los factores comunes tienen signos opuestos (se actualiza el signo del módulo de λ_s).

$$J_X = |\mathrm{U}| * \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & \overset{*}{1} & 0 & 0 & 0 & 0 & 0 & 0 & \overset{*}{0} & \overset{*}{0} & 0 \\ 0 & 0 & 0 & 0 & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad J_O = |\mathrm{U}| * \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$|\mathrm{U}|$: Matriz Unitaria.

Se separan los elementos correspondientes a fuerzas y momentos (*):

$$J_x = \begin{vmatrix} J_{F_x} & 0 \\ 0 & J_{M_x} \end{vmatrix} = |\mathrm{U}| * \begin{vmatrix} \begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix} & \varnothing \\ \varnothing & \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ \bar{1} & \bar{1} & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{matrix} \end{vmatrix}$$

$$J_x = \begin{vmatrix} J_{F_o} & 0 \\ 0 & J_{M_o} \end{vmatrix} = |U| * \left| \begin{array}{ccccccc|c} 1 & 1 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & \varnothing \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \end{array} \right|$$

7.4

$$\Lambda_X = \begin{vmatrix} \Lambda_{F_x} & 0 \\ 0 & \Lambda_{M_X} \end{vmatrix} = \left\{ \lambda_{y_A} \lambda_{x_B} \lambda_{y_B} \lambda_{z_B} \lambda_{x_C} \lambda_{y_C} \lambda_{z_C} \lambda_{x_D} \lambda_{y_D} \lambda_{z_D} \lambda_{x_E} \lambda_{y_E} \lambda_{z_E} \,\vdots\, \lambda_{m_{ZC}} \lambda_{m_{XD}} \lambda_{m_{XE}} \lambda_{m_{YE}} \lambda_{m_{ZE}} \right\}$$

$$= \left\{ \begin{array}{ccccccccccccc|ccccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right\}$$

$$\Lambda_O = \begin{vmatrix} \Lambda_{F_O} & 0 \\ 0 & \Lambda_{M_O} \end{vmatrix} = \left\{ \lambda_{P_1} \lambda_{P_2} \lambda_{P_3} \lambda_{P_4} \lambda_{P_5} \lambda_{P_6} \lambda_{P_7} \right\} = \left\{ \begin{array}{ccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & \bar{1} & \bar{1} & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right\}$$

7.5 Para momentos en G (0, 1, 3)

$$\Delta_X = \left\{ \Delta_{Y_A} \Delta_{X_B} \Delta_{Y_B} \Delta_{Z_B} \Delta_{X_C} \Delta_{Y_C} \Delta_{Z_C} \Delta_{X_D} \Delta_{Y_D} \Delta_{Z_D} \Delta_{X_E} \Delta_{Y_E} \Delta_{Z_E} \right\}$$

$$= \left\{ \begin{vmatrix} 0-0 \\ 0-1 \\ 0-3 \end{vmatrix} \begin{vmatrix} 2-0 \\ 1-1 \\ 0-3 \end{vmatrix} \begin{vmatrix} 2 \\ 0 \\ \bar{3} \end{vmatrix} \begin{vmatrix} 2 \\ 0 \\ \bar{3} \end{vmatrix} \begin{vmatrix} 0-0 \\ 1-1 \\ 2-3 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} 1-0 \\ 1-1 \\ 3-3 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} 3-0 \\ 1-1 \\ 3-3 \end{vmatrix} \begin{vmatrix} 3 \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} 3 \\ 0 \\ 0 \end{vmatrix} \right\}$$

$$= \left\{ \begin{array}{ccccccccccccc} 0 & 2 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 3 & 3 & 3 \\ \bar{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{3} & \bar{3} & \bar{3} & \bar{3} & \bar{1} & \bar{1} & \bar{1} & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right\}$$

$$\Delta_0=\left\{\Delta_{P_1}\Delta_{P_2}\Delta_{P_3}\Delta_{P_4}\Delta_{P_5}\Delta_{P_6}\Delta_{P_7}\right\}=\left\{\begin{Vmatrix}0-0\\1-1\\0-3\end{Vmatrix}\begin{Vmatrix}1-0\\1-1\\0-3\end{Vmatrix}\begin{Vmatrix}0-0\\1-1\\2-3\end{Vmatrix}\begin{Vmatrix}0-0\\1-1\\\frac{5}{2}-3\end{Vmatrix}\begin{Vmatrix}0-0\\1-1\\3-3\end{Vmatrix}\begin{Vmatrix}\frac{1}{2}-0\\1-1\\3-3\end{Vmatrix}\begin{Vmatrix}0-0\\1-1\\3-3\end{Vmatrix}\right\}$$

$$=\left\{\begin{matrix}0&1&0&0&0&\frac{1}{2}&1\\0&0&0&0&0&0&0\\\bar{3}&\bar{3}&\bar{1}&\overline{\frac{1}{2}}&0&0&0\end{matrix}\right\}$$

7.6

$$W_x=I*\Delta_x=\left\|\begin{matrix}0&0&0\\0&0&1\\0&\bar{1}&0\end{matrix}\middle|\begin{matrix}0&0&\bar{1}\\0&0&0\\1&0&0\end{matrix}\middle|\begin{matrix}0&1&0\\\bar{1}&0&0\\0&0&0\end{matrix}\right\|*\left\{\begin{Vmatrix}0&2&2&2&0&0&0&1&1&1&3&3&3\\\bar{1}&0&0&0&0&0&0&0&0&0&0&0&0\\\bar{3}&\bar{3}&\bar{3}&\bar{3}&\bar{1}&\bar{1}&\bar{1}&0&0&0&0&0&0\end{Vmatrix}\right\}$$

$$=-\left\{\begin{Vmatrix}0&\bar{3}&\bar{1}\\3&0&0\\\bar{1}&0&0\end{Vmatrix}\begin{Vmatrix}0&\bar{3}&0\\3&0&2\\0&\bar{2}&0\end{Vmatrix}\begin{Vmatrix}0&\bar{3}&0\\3&0&2\\0&\bar{2}&0\end{Vmatrix}\begin{Vmatrix}0&\bar{3}&0\\3&0&2\\0&\bar{2}&0\end{Vmatrix}\begin{Vmatrix}0&\bar{1}&0\\1&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&\bar{1}&0\\1&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&\bar{1}&0\\1&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&1\\0&\bar{1}&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&1\\0&\bar{1}&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&1\\0&\bar{1}&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&3\\0&\bar{3}&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&3\\0&\bar{3}&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&3\\0&\bar{3}&0\end{Vmatrix}\right\}$$

$$W_x=I*\Delta_x=\left\|\begin{matrix}0&0&0\\0&0&1\\0&\bar{1}&0\end{matrix}\middle|\begin{matrix}0&0&\bar{1}\\0&0&0\\1&0&0\end{matrix}\middle|\begin{matrix}0&1&0\\\bar{1}&0&0\\0&0&0\end{matrix}\right\|*\left\{\begin{vmatrix}0\\0\\\bar{3}\end{vmatrix}\begin{vmatrix}1\\0\\\bar{3}\end{vmatrix}\begin{vmatrix}0\\0\\\bar{1}\end{vmatrix}\begin{vmatrix}0\\0\\\overline{\frac{1}{2}}\end{vmatrix}\begin{vmatrix}0\\0\\0\end{vmatrix}\begin{vmatrix}\frac{1}{2}\\0\\0\end{vmatrix}\begin{vmatrix}1\\0\\0\end{vmatrix}\right\}$$

$$=-\left\{\begin{Vmatrix}0&\bar{3}&0\\3&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&\bar{3}&0\\3&0&1\\0&\bar{1}&0\end{Vmatrix}\begin{Vmatrix}0&\bar{1}&0\\1&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&\overline{\frac{1}{2}}&0\\\frac{1}{2}&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&\frac{1}{2}\\0&\overline{\frac{1}{2}}&0\end{Vmatrix}\begin{Vmatrix}0&0&0\\0&0&1\\0&\bar{1}&0\end{Vmatrix}\right\}$$

7.7

$$K_x = \left[\; |\mathrm{U}| * \left|\begin{array}{ccccccccccccc|ccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & 0 & 0 & 0 & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & \bar{1} & 0 & 0 & 0 & & & \varnothing & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & & & & & \\
\hline
 & & & & & & & & & & & & & 1 & 0 & 0 & 0 & 0 \\
 & & & & & & \varnothing & & & & & & & \bar{1} & \bar{1} & 0 & 0 & 0 \\
 & & & & & & & & & & & & & 0 & 1 & 1 & 1 & 1
\end{array}\right| * \right.$$

$$\left[\; \left|\begin{array}{c|c}
\mathrm{U} & \varnothing \\
\hline
\left\{\left|\begin{array}{ccc|ccc|ccc|ccc|ccc|ccc|ccc|ccc|ccc|ccc|ccc|ccc|ccc}
0 & 3 & \bar{1} & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{3} & 0 & 0 & \bar{3} & 0 & \bar{2} & \bar{3} & 0 & \bar{2} & \bar{3} & 0 & \bar{2} & \bar{1} & 0 & 0 & \bar{1} & 0 & 0 & \bar{1} & 0 & 0 & 0 & 0 & \bar{1} & 0 & 0 & \bar{1} & 0 & 0 & \bar{1} & 0 & 0 & \bar{3} & 0 & 0 & \bar{3} & 0 & 0 & \bar{3} \\
\bar{1} & 0 & 0 & 0 & 2 & 0 & 0 & \bar{2} & 0 & 0 & \bar{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0
\end{array}\right|\right\} & \mathrm{U}
\end{array}\right| * \right.$$

$$\left. * \left|\left\{\begin{array}{ccccccccccccc|ccccc}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}\right\}\right|^{\downarrow} \;\right]^{\downarrow}$$

$$K_x = \left| \begin{array}{cccccccccccccccccc}
0 & 1 & 0 & 0 & 1 & 0 & 0 & & & & & & & & & & & \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & & & & & & & & & & & \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & & & & & & & & & & & \\
 & & & & \bar{1} & 0 & 0 & \bar{1} & 0 & 0 & & & & & & & & \\
 & & & & 0 & \bar{1} & 0 & 0 & \bar{1} & 0 & & & & & & & & \\
 & & & & 0 & 0 & \bar{1} & 0 & 0 & \bar{1} & & & & & & & & \\
 & & & & & & & 1 & 0 & 0 & 1 & 0 & 0 & & & & & \\
 & & & & & & & 0 & 1 & 0 & 0 & 1 & 0 & & & & & \\
 & & & & & & & 0 & 0 & 1 & 0 & 0 & 1 & & & & & \\
 & & & & & & & & & & & & & & & & & \\
3 & 0 & 3 & 0 & 0 & 1 & 0 & & & & & & & 0 & & & & \\
0 & \bar{3} & 0 & \bar{2} & \bar{1} & 0 & 0 & & & & & & & 0 & & & & \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & & & & & & & 1 & & & & \\
 & & & & 0 & \bar{1} & 0 & 0 & 0 & 0 & & & & 0 & \bar{1} & & & \\
 & & & & 1 & 0 & 0 & 0 & 0 & 1 & & & & 0 & 0 & & & \\
 & & & & 0 & 0 & 0 & 0 & \bar{1} & 0 & & & & \bar{1} & 0 & & & \\
 & & & & & & & 0 & 0 & 0 & 0 & 0 & 0 & & 1 & 1 & 0 & 0 \\
 & & & & & & & 0 & 0 & \bar{1} & 0 & 0 & \bar{3} & & 0 & 0 & 1 & 0 \\
 & & & & & & & 0 & 1 & 0 & 0 & 3 & 0 & & 0 & 0 & 0 & 1
\end{array} \right|$$

7.8

$$K_o = \left[\begin{array}{c|ccccccc}
 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
|U| \quad * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
 & & & & \varnothing & & &
\end{array} \right|^{*}$$

$$\left[\begin{array}{c|c} \begin{array}{c} \mathrm{U} \\ \hline \left\{\left\|\begin{array}{ccc|ccc|ccc|ccc|ccc|ccc|ccc} 0 & 3 & 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{3} & 0 & 0 & \bar{3} & 0 & 1 & \bar{1} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{\tfrac{1}{2}} & 0 & 0 & \bar{1} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 1 & 0 \end{array}\right\|\right\} \end{array} & \begin{array}{c} \varnothing \\ \hline \mathrm{U} \end{array} \end{array}\right| * \left\|\left\{\begin{array}{ccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & \bar{1} & \bar{1} & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right\}\right\|^{\downarrow}\Bigg]^{\downarrow}$$

$$K_o = \left|\begin{array}{ccccccc} 0 & 0 & & & & & \\ 0 & 0 & & & & & \\ 1 & 1 & & & & & \\ & & 1 & 0 & 0 & 0 & 0 \\ & & 0 & \bar{1} & \bar{1} & \bar{1} & 0 \\ & & 0 & 0 & 0 & 0 & 1 \\ & & & & & & 0 \\ & & & & & & 0 \\ & & & & & & 0 \\ 0 & 0 & & & & & \\ 0 & \bar{1} & & & & & \\ 0 & 0 & & & & & \\ & & 1 & \bar{1}/2 & 0 & 0 & 0 \\ & & \bar{1} & 0 & 0 & 0 & \bar{1} \\ & & 0 & 0 & 0 & 1/2 & 0 \end{array}\right|$$

$$
K_X^{-1} = \begin{array}{c|cccccccccccccccccc|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
Y_A & & \tfrac{1}{4} & & & \tfrac{1}{2} & & & & & \tfrac{1}{4} & & \overline{\tfrac{1}{2}} & & & \overline{\tfrac{1}{2}} & & & \\
X_B & \tfrac{1}{4} & & \overline{\tfrac{1}{2}} & & & \overline{\tfrac{1}{2}} & & & & & \overline{\tfrac{1}{4}} & & & \overline{\tfrac{1}{2}} & & & & \\
Y_B & & \overline{\tfrac{3}{4}} & & & \overline{\tfrac{1}{2}} & & & & & \tfrac{1}{4} & & \tfrac{1}{2} & & & \tfrac{1}{2} & & & \\
Z_B & \overline{\tfrac{3}{4}} & & \tfrac{1}{2} & & & \tfrac{1}{2} & & & & & \overline{\tfrac{1}{4}} & & & \tfrac{1}{2} & & & & \\
X_C & \tfrac{3}{4} & & \tfrac{1}{2} & & & \tfrac{1}{2} & & & & & \tfrac{1}{4} & & & \tfrac{1}{2} & & & & \\
Y_C & & \tfrac{3}{2} & & & & & & & & \overline{\tfrac{1}{2}} & & & & & & & & \\
Z_C & \tfrac{3}{4} & & \tfrac{1}{2} & & & \overline{\tfrac{1}{2}} & & & & & \tfrac{1}{4} & & & \overline{\tfrac{1}{2}} & & & & \\
X_D & \overline{\tfrac{3}{4}} & & \overline{\tfrac{1}{2}} & \bar{1} & & \overline{\tfrac{1}{2}} & & & & & \overline{\tfrac{1}{4}} & & & \overline{\tfrac{1}{2}} & & & & \\
Y_D & & \overline{\tfrac{3}{2}} & & & \bar{1} & & & & & \tfrac{1}{2} & & & & & & & & \\
Z_D & \overline{\tfrac{3}{4}} & & \overline{\tfrac{1}{2}} & & & \overline{\tfrac{1}{2}} & & & & & \overline{\tfrac{1}{4}} & & & \tfrac{1}{2} & & & & \\
X_E & \tfrac{3}{4} & & \tfrac{1}{2} & 1 & & \tfrac{1}{2} & 1 & & & & \tfrac{1}{4} & & & \tfrac{1}{2} & & & & \\
Y_E & & \tfrac{3}{2} & & & 1 & \tfrac{1}{2} & & 1 & & \overline{\tfrac{1}{2}} & & & & & & & & \\
Z_E & \tfrac{3}{4} & & \tfrac{1}{2} & & & & & & 1 & & \tfrac{1}{4} & & & \overline{\tfrac{1}{2}} & & & & \\
M_{ZC} & & \tfrac{3}{2} & & & 1 & & & & & \overline{\tfrac{1}{2}} & & & & & \bar{1} & & & \\
M_{XD} & & \tfrac{3}{2} & & & & & & & & \overline{\tfrac{1}{2}} & & & \bar{1} & & & & & \\
M_{XE} & & \tfrac{3}{2} & & & & & & & & \overline{\tfrac{1}{2}} & & & 1 & & & 1 & & \\
M_{YE} & \tfrac{3}{2} & & 1 & & & 1 & & & 3 & & \tfrac{1}{2} & & & \bar{1} & & & 1 & \\
M_{ZE} & & \bar{3} & & & \bar{2} & & & \bar{3} & & 1 & & & & & & & & 1
\end{array}
$$

$$
Q_x = -K_x^{-1} K_0 Q_0 = -K_x^{-1} \left|\begin{array}{ccccccc}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 \\
0 & & & & & & \\
0 & & & & & & \\
1 & 1 & & & & & \\
0 & & 1 & & & & \\
0 & & & \bar{1} & \bar{1} & \bar{1} & \\
0 & & & & & & 1 \\
0 & & & & & & \\
0 & & & & & & \\
0 & & & & & & \\
0 & & & & & & \\
0 & \bar{1} & & & & & \\
0 & & & & & & \\
0 & & & \overline{\tfrac{1}{2}} & & & \\
0 & & \bar{1} & & & & \bar{1} \\
0 & & & & & \overline{\tfrac{1}{2}} & \\
0 & & & & & & \\
0 & & & & & & \\
0 & & & & & &
\end{array}\right| \left|\begin{array}{c} 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 2 \end{array}\right|
$$

$$
= - \begin{vmatrix}
 & & & \overline{\tfrac{1}{2}} & \overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \\
\overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \tfrac{1}{2} & & & & \\
 & & & \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{4} & \\
\tfrac{1}{2} & \tfrac{3}{4} & \overline{\tfrac{1}{2}} & & & & \\
\tfrac{1}{2} & \tfrac{1}{4} & \overline{\tfrac{1}{2}} & & & & \\
0 & & & & & & \\
\tfrac{1}{2} & \tfrac{1}{4} & \tfrac{1}{2} & & & & \\
\overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \overline{\tfrac{1}{2}} & & & & \\
 & & & 1 & 1 & 1 & \\
\overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \overline{\tfrac{1}{2}} & & & & \bar{1} \\
\tfrac{1}{2} & \tfrac{1}{4} & \tfrac{1}{2} & & & & \\
 & & & \bar{1} & \bar{1} & \bar{1} & \\
\tfrac{1}{2} & \tfrac{1}{4} & \tfrac{1}{2} & & & & 1 \\
 & & & \bar{1} & \bar{1} & \overline{\tfrac{1}{2}} & \\
 & & & \tfrac{1}{2} & & & \\
 & & & \tfrac{1}{2} & & & \\
1 & \tfrac{1}{2} & 1 & & & & 2 \\
 & & & 2 & 2 & 2 &
\end{vmatrix}
\begin{Vmatrix}
1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 2
\end{Vmatrix}
$$

$$
= - \begin{vmatrix}
 & & & \overline{\tfrac{1}{2}} & \overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \\
\overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & 1 & & & & \\
 & & & \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{4} & \\
\tfrac{1}{2} & \tfrac{3}{4} & \bar{1} & & & & \\
\tfrac{1}{2} & \tfrac{1}{4} & \bar{1} & & & & \\
0 & & & & & & \\
\tfrac{1}{2} & \tfrac{1}{4} & 1 & & & & \\
\overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \bar{1} & & & & \\
 & & & 1 & 1 & 1 & \\
\overline{\tfrac{1}{2}} & \overline{\tfrac{1}{4}} & \bar{1} & & & & \bar{2} \\
\tfrac{1}{2} & \tfrac{1}{4} & 1 & & & & \\
 & & & \bar{1} & \bar{1} & \bar{1} & \\
\tfrac{1}{2} & \tfrac{1}{4} & 1 & & & & 2 \\
 & & & \bar{1} & \bar{1} & \overline{\tfrac{1}{2}} & \\
 & & & \tfrac{1}{2} & & & \\
 & & & \tfrac{1}{2} & & & \\
1 & \tfrac{1}{2} & 2 & & & & 4 \\
 & & & 2 & 2 & 2 &
\end{vmatrix}
=
\begin{vmatrix}
\overline{\tfrac{5}{4}} \\ \overline{\tfrac{1}{4}} \\ \tfrac{5}{4} \\ \tfrac{1}{4} \\ \tfrac{1}{4} \\ 0 \\ \overline{\tfrac{7}{4}} \\ \tfrac{7}{4} \\ \bar{3} \\ \tfrac{15}{4} \\ \overline{\tfrac{7}{4}} \\ 3 \\ \overline{\tfrac{15}{4}} \\ \tfrac{5}{2} \\ \overline{\tfrac{1}{2}} \\ \tfrac{1}{2} \\ \overline{\tfrac{15}{2}} \\ \bar{6}
\end{vmatrix}
$$